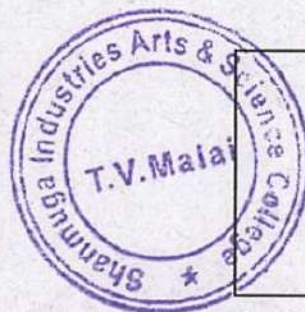


SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.



APRIL/MAY 2025

GMA13/DMA13 — ORDINARY
DIFFERENTIAL EQUATION

Time : Three hours

Maximum : 75 marks

16. Solve : $y'' + 2iy' + y = x$.
17. Solve : $y''' + y'' + y = 1$ which satisfies $\psi''(0) = 0, \psi''(0) = 1, \psi''(0) = 0$.
18. Find two linearly independent power series solution of $y'' - xy = 0$.
19. Find the solution of the equation $x^2 + y'' + \frac{3}{2}xy' + xy = 0$.
20. Prove that the k^{th} successive approximation ϕ_k to the solution ϕ of the initial value problem $y' = f(x, y), y(x_0) = 0$ satisfies

$$|\phi(x) - \phi_k(x)| \leq \frac{M}{K} \frac{(k\alpha)^{k+1}}{(k+1)!} e^{k\alpha}, \text{ for all } x \text{ in } I.$$

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define non-homogeneous equation with an example.
2. Define wronukian of ϕ_1, ϕ_2 .
3. Define initial value problem for n^{th} order equation.
4. State the uniqueness theorem for n^{th} order equation.
5. Define Gramian of $\phi_1, \phi_2, \dots, \phi_n$.
6. Write Legendre equation.
7. Write the indicial polynomial for $x^2y'' + a(n)xy' + b(x)y = 0$.

8. Write the Bessel function of order n of the second kind.
9. Find the real valued solutions of the equation $y' = x^2 y$.
10. State Lipschitz condition.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.



11. (a) Find all solutions of the equation $y'' - 4y' + 5y = 0$.

Or

- (b) Solve $y'' + 4y = \cos n$.

12. (a) Find all solution of the equation $y''' - 3y' - 2y = 0$.

Or

- (b) Using the annihilator method find a particular solution of equation $y''' = x^2 + e^{-x} \sin x$.

13. (a) Solve : $(3x-1)^2 y'' + (9x-3)y' - 9y = 0$.

Or

- (b) Show that $\int_{-1}^1 p n^2(x) dx = \frac{2}{2n+1}$.

14. (a) Solve : $x^2 y'' + xy' - 4y = x; x > 0$.

Or

- (b) Show that $x^{\frac{1}{2}} \sqrt{\frac{1}{2}}(x) = \frac{\sqrt{2}}{\sqrt{\frac{1}{2}}} \sin x$.

15. (a) Show that the solution of $y^1 = y^2$ which passes through the point (x_0, y_0) is given by

$$\phi(x) \frac{y_0}{1 - y_0(x - x_0)}.$$

Or

- (b) Solve : $(e^y + xe^y)dx + xe^y dy = 0$.